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$$\dot{p} = (c_1 r + c_2 p)q + c_3 \overline{L} + c_4 N$$

$$\dot{q} = c_5 pr - c_6 (p^2 - r^2) + c_7 M$$

$$\dot{r} = (c_8 p - c_2 r)q + c_4 \overline{L} + c_9 N$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \qquad (15)$$

$$\dot{\psi} = \frac{q\sin\phi + r\cos\phi}{\cos\theta}$$

$$\dot{x}_E = u\cos\theta\cos\psi + v(-\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi) + \\ + w(\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi)$$

$$\dot{y}_E = u\cos\theta\sin\psi + v(\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi) + \\ + w(-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi)$$

$$\dot{h}_E = -u\sin\theta + v\sin\phi\cos\theta + w\cos\phi\cos\theta$$

The state vector for the system of Equations (15) is :

$$\underline{X}^T = [u \quad v \quad w \quad \phi \quad \theta \quad \psi \quad p \quad q \quad r \quad x_E \quad y_E \quad h_E]$$

with the distance from ground (h) instead of the altitude z.
By using the above described mathematical model of Aerodynamic forces and moments it is possible to solve the collision avoidance problem.
The best strategy is to maximize wrt the controls the timewise minimum distance between the aircraft and the obstacle. At the maximin point of the encounter, the distance between the UAV and the obstacle has a minimum wrt the time, which occurs when the relative position vector is orthogonal to the relative velocity vector. In this way, we obtain an inner boundary condition to be satisfied at the maximin point separating the two main branches of the maneuver: the avoidance branch and the recovery branch. As a consequence, a one-subarc Chebyshev problem wich is not solvable in a direct way can be transformed into a two-subarc Bolza problem solvable via the multiple-subarc sequential gradient-restoration algorithm (SGRA)[18],[19].

4 Conclusion

A non-linear model has been built in order to model the aerodynamic characteristics of an UAV flying In Ground Effect. The obtained model can be used in the whole range of flying altitude,because of the aerodynamic coefficients depend, in a direct way, on the ground distance. Such a model has been employed to describe aerodynamic forces and moments in the equations of motion. To cope with the Collision Avoidance problem it has been proposed to transform the Chebyshev problem of optimal control into a Bolza problem, which can be solved , numerically, applying the multiple-subarc sequential gradient-restoration algorithm

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$$\Delta\alpha_{IGE} = \left(\frac{3.3574 + 0.0045 \cdot \left(\frac{h}{b}\right)^{-1.748}}{3.3574} - 1 \right) \cdot \left(1 + 0.001317 \left(\frac{h}{b}\right)^{-1.75} \right) \cdot \alpha_{OGE} \qquad (6)$$

$$\Delta\varepsilon_g = \varepsilon \frac{b_{eff}^2 + 4(H_h - H_w)^2}{b_{eff}^2 + 4(H_h + H_w)^2} \qquad (7)$$

where b^2_{eff} is the wing span in IGE and it has been evaluated by using calassical methodologies[16], [17].

Moreover:

$$H_w = \frac{h}{b} b - h_{nb} \sin\alpha_g$$
$$H_h = \frac{h}{b} b + h_{nf} \sin\alpha_g \qquad (8)$$

In order to evaluate the aspect ratio variation due to altitude, the variation of the induced angle of attack caused by the altitude has been evaluated for each wing:

$$-\frac{1}{\pi\lambda_e} = \frac{c_{L\alpha_{IGE}}}{c_{L\alpha_{OGE}}} \cdot \frac{1}{c_{L\alpha_{airfoil}}} \left(1 - \frac{c_{L\alpha_{airfoil}}}{\pi\lambda_{OGE}} \right) - \frac{1}{c_{L\alpha_{airfoil}}} \qquad (9)$$

2.2 Lateral Coefficients

The variation of either Side Force, or Rolling and Yawing Moment due to Ground Effect have been obtained taking into account :

- bank angle(ϕ) effects
- angle of attack (α) variation on aerodynamic surfaces
- aspect ratio(λ) variation due to flight altitude

Side force has been evaluated by modeling the variation of the angle of attack (α) of the vertical tail [16] :

$$\Delta\alpha_g = -18.24(C_L)\frac{\sigma_g}{\lambda} + \frac{r_g T_g (C_L)^2}{57.3 C_{L\alpha}} - r_g B_g \qquad (10)$$

with:

$$\sigma_g = e^{-2.48\left(\frac{h}{b/2}\right)^{0.768}} \qquad (11)$$

$$r_g = \sqrt{1 + \left(\frac{h}{b/2}\right)^2} - \frac{h}{b/2} \qquad (12)$$

$$T_g = \frac{1}{8\pi} \frac{\frac{h}{c}}{\left(\frac{h}{c}\right)^2 + \frac{1}{64}} \qquad (13)$$

To evaluate B_g the methodology of [16] has been used.

Rolling and Yawing Moments have been calculated, by inserting Eqs (6), (7), (9) into Eqs (1) (2),(4), (5), and by modeling the ϕ effects.

In fact, near the ground, left and right wing have different distance from the solid surface, so ϕ generates an angle of attack variation .

In particular, for a positive bank angle, the right(h_r) and left (h_l) wing distances from the ground are:

$$h_r = h + y \sin\phi$$
$$h_l = h - y \sin\phi \qquad (14)$$

(h is the distance from the ground of the center of gravity).

By inserting Eq.(14) into Eqs (6), (7), (9), it is possible to model lift and drag to evaluate rolling and yawing moments.

3 Collision Avoidance problem

As it is well known, the dynamic equations of a rigid UAV, flying over a flat Heart , in body axes are:

$$\dot{u} = rv - qw - g\sin\theta + \frac{F_{X,B}}{m}$$
$$\dot{v} = -ru + pw + g\sin\phi\cos\theta + \frac{F_{Y,B}}{m}$$
$$\dot{w} = qu - pv + g\cos\phi\cos\theta + \frac{F_{Z,B}}{m}$$

take-off and or landing manoeuvres, for example, the aerodynamic coefficients are modelled by using semi-empirical equations Often, medium values of these ones are used . Nevertheless, stability characteristics are not determined in the whole range of the h/b *ratio* (altitude-span ratio) from in to out ground effect.. Therefore, up to now, two different mathematical models are used to study the dynamic behaviour of aircraft out or in ground effect, [7],[9], [10],[11],[12].
In the present paper, a general mathematical model is built to obtain non – linear analytical equations for aerodynamic coefficients both Out of Ground Effect and In Ground Effect conditions. Such a mathematical model is used to obtain optimal trajectories for solving the collision avoidance problem for a particular UAV in Tandem – Canard architecture [5], [13].

2 Mathematical Model for Aerodynamic coefficients

The studied UAV, as previous stated, has a tandem – canard arrangement; it is particularly suitable to very low altitude missions (for example forest fire detection, volcanoes monitoring and/or battle field surveillance). The fixed tail surface (S_f)is identical to the main wing(S_b) .

All the aerodynamic coefficients have been evaluated in function of classical variables and including also the flight altitude. Obviously, because of the strong nonlinearities due to the ground effect presence, the superposition principle is not applied.

2.1 Longitudinal Coefficients

The influence of ground effect on aerodynamic coefficient has been evaluated as an angle of attack (α) variation, as a downwash(ε) variation and as an aspect ratio(λ) variation due to flight altitude [6], [14]. So, the equations of Lift, Drag and Pitching Moment are:

$$L=\frac{1}{2}\rho V_w^2 S\left(\frac{1}{2}c_{Lf}+\frac{1}{2}c_{Lb}\cdot\cos\varepsilon\right) \quad (1)$$

$$D=\frac{1}{2}\rho V_w^2 S\begin{pmatrix} c_{D0}+\Delta c_{D0}\left(\delta_{el}\right)+ \\ +\frac{1}{2}\frac{c_{Lf}^2}{\pi\lambda_f}+\frac{1}{2}\frac{c_{Lb}^2}{\pi\lambda_b}\cos\varepsilon \end{pmatrix} \quad (2)$$

$$\begin{aligned} M = &\frac{1}{2}\rho V_w^2 Sc\Bigg(-\frac{1}{2}\frac{h_{nb}}{c}\Big(c_{Lb}\cos\left(\alpha_g-\varepsilon\right)+ \\ &+\left(\frac{1}{2}c_{D0}+\frac{c_{Lb}^2}{\pi\lambda_b}\right)\sin\left(\alpha_g-\varepsilon\right)\Big)+ \\ &+\frac{1}{2}\frac{h_{nf}}{c}\Big(c_{Lf}\cos\left(\alpha_g\right)+ \\ &\left(\frac{1}{2}c_{D0}+\frac{c_{Lf}^2}{\pi\lambda_f}\right)\sin\left(\alpha_g\right)\Big)-c_{m0}\Bigg)\end{aligned} \quad (3)$$

Obviously, c_{Lf} and c_{Lb} (respectively forward and backward wing lift coefficient) depend on angle of attack of the single wing:

$$\alpha_f=i_t+\arctan\left(\frac{w-q\cdot h_{nf}}{u}\right)+\Delta\alpha_{IGE} \quad (4)$$

$$\begin{aligned}\alpha_b=&\arctan\left(\frac{w+q\cdot h_{nf}}{u}\right)- \\ &+\left(\varepsilon-\left(\frac{\partial\varepsilon}{\partial\alpha}\bigg|_{OGE}\left(1-\Delta\varepsilon_{IGE}\right)\cdot 4q\frac{h_{nf}+h_{nb}}{V_w}\right)\right)+\Delta\alpha_{IGE}\end{aligned} \quad (5)$$

$\Delta\alpha_{IGE}$ is the angle of attack variation due to ground distance.

In a previous research [15], it has been found that aerodynamic coefficients can be expressed by hyperbolic equations. These ones afford to evaluate aerodynamic coefficients both in Out Ground Effect (OGE) and In Ground Effect (IGE) conditions.

Mathematical Model of Unmanned Aircraft In Ground Effect for Optimal Collision Avoidance

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Keywords: *Ground Effect, UAS, Non Linear Mode,*

Abstract

The fundamental aim of the present paper is to model the aerodynamic characteristics of an Unmanned Aerial Vehicle(UAV) flying in ground effect. The second aim is to determine the relationship between the optimal avoidance maneuver and the control to execute it. In fact, this relationship is basilar to the development of a guidance scheme capable of approximating the optimal trajectory in real time. In the first part of this work ,a non-linear model is built in order to model the aerodynamic characteristics of an UAV flying In Ground Effect. To use a single model in the whole range of flying altitude, aerodynamic coefficients are modeled by means of hyperbolic equations. In particular, these ones depend on the ground distance. Such a model is employed to describe aerodynamic forces and moments in the equations of motion which are used in order to obtain optimal trajectories for solving the collision avoidance problem.

1 General Introduction

As it is well known, UAVs are today employed for several civil and military missions, such as territorial surveillance (forest fire detection, volcanoes monitoring, etc.), law enforcement, disaster assistance, frontier surveillance, agricultural surveying, power-line monitoring, archaeological sites surveillance and many others [[1]], [[2]], [[3]]. In particular, they can be usefully employed in applications which require flight in ground effect (IGE) above water surface [[4]]. This is the case, for instance, of antisubmarine missions [[5]] and, in the civil field, of search and rescue operations or servicing and maintenance of oil rigs. as a consequence, they need to operate at very low altitude, under the influence of extreme ground effect [1], [2], [3]. Safe IGE flight requires high performance flight control systems, particularly when dealing with relatively small aircraft (and this is often the case for UAVs) and/or flight above rough sea.

As it is known, when an aircraft flies near the ground, the lift increases, the induced drag decreases, the neutral point shifts, the pitching moment at zero lift varies and it is necessary to take into account the bank angle(ϕ) derivatives[6].

. The stability characteristics of such a vehicle are strongly correlated to the ground distance, so the primary goal of researchers in this field is the evaluation of aerodynamic coefficients in presence of ground effect. Many Authors have carried out studies in this field[7],[8],[9], [10].In spite of all these studies, there are no mathematical model which affords to analyze the whole flight range of an aircraft. To study